GRADUATION OF THE 1985-90 ASSURED LIFE MORTALITY EXPERIENCE: A QUESTION OF MAINTAINING STANDARDS.

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ABSTRACT

This paper examines the reasonableness of producing a non-racial aggregated standard mortality table for South African assured lives based on the experience over the period 1985-90. It finds that not only are the mortality rates between different sub-groups significantly different but that the rates of growth in the number of new policies in some of the sub-groups also differ significantly, and hence concludes that it is not appropriate to produce a single standard life table for all assured lives. The paper concludes with a number of questions for the profession practising in an environment of extreme heterogeneity and worsening aggregate mortality.

1. INTRODUCTION

For some time now various members of the Actuarial Society (in particular members of the CSI Committee) have suggested that there is a need for a new standard table. Some of the reasons given are:

1. 1956-62 was a long time ago and there is evidence of secular improvements in the ultimate mortality since the 1972/77 investigation (CSI Committee, 1994) - and the select mortality appears to have worsened significantly. In addition the 1956/62 "accident hump" at the younger ages was purposefully removed.

2. The 1985-90 data represents, probably, the last experience which will have negligible AIDS related deaths, and thus can provide some sort of benchmark.
3. It is no longer appropriate (politically nor practically) to base the standard mortality on the White Male lives alone. Indeed most offices now no longer segregate their experience on the basis of race and the CSI Committee no longer collects data in this format.

Thus the purpose of this paper is to graduate the 1985-90 data with a view to producing a new standard table based on all male assured lives combined experience.

2. THE DATA

The data used in this exercise was the data submitted to the CSI committee by contributing offices for South African whole life and endowment assured lives in force over the period 1985-90. The data was collected in two batches, 1985-86, and 1987-90. SANLAM, Old Mutual, Fedlife, Liberty Life and Metropolitan contributed to both periods while, Colonial, Norwich and Traduna contributed to the first period only. Although this means that fewer than half of the life offices in South Africa participated in the investigation the data probably represents over 80% of the industry’s exposure.

The data was separated by racial classification, sex and, for White male and female lives, according to whether or not they underwent a medical examination prior to being accepted at standard rates. Although some contributors were unable to provide data divided by racial classification the effect was thought not to be significant.

It is obviously vitally important that the data be carefully validated. However, the data available to the authors was available only in aggregate form which made it impossible for the authors to check individual company submissions and we had to assume that the necessary validation checks had been carried out by the office which collates the data. Inspection of the aggregate data did not reveal any oddities which could be specifically linked to a lack of appropriate validation.

Although over the years it has been suggested (e.g. Mortality Standing Committee, 1983 and CSI Committee, 1994) that selective withdrawals (particularly those caused by lumpsum benefits payable on disability) could, increasingly, be leading to an understatement of mortality, data was not available to enable us to investigate this hypothesis. (However, the 1991-94 data (CSI Committee, 1996) show that although the mortality of lives with policies that have "accelerator benefits" is indeed lighter - suggesting possible selective withdrawals - this effect on aggregate mortality is likely to be fairly limited since there are very few of these policies at the older ages.)

The data included duplicate policies and although there was no way of identifying the extent of multiple policies we attempted to allow for them in estimating the confidence limits by, somewhat arbitrarily, dividing the deaths and exposed to risk by smoothed "variance ratios" derived from those produced as part of the graduation of the 1979-82 UK life assurance experience (Forfar, et al, 1988: 121).

A summary of the data available for the period 1985-90 appears in Appendix 1 (greater detail can be found in Rosenberg, 1996). Table 1 below compares this experience with that available for the 1956/62 and 1972/77 investigations.

Table 1: RELATIVE EXPOSED TO RISK FOR MALE ULTIMATE LIVES

<table>
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<tr>
<th>Age</th>
<th>SA56/62 Exposed to Risk (000s)</th>
<th>SA72/77</th>
<th>1985-90 White Males</th>
<th>1985-90 All Males</th>
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<td>15-19</td>
<td>26.1</td>
<td>1.20</td>
<td>1.14</td>
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<td>20-29</td>
<td>746.9</td>
<td>1.17</td>
<td>1.12</td>
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<td>30-59</td>
<td>3 687.6</td>
<td>1.33</td>
<td>1.63</td>
<td>1.96</td>
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<td>60+</td>
<td>377.7</td>
<td>1.69</td>
<td>2.38</td>
<td>2.51</td>
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<tr>
<td>Total</td>
<td>4 838.3</td>
<td>1.33</td>
<td>1.60</td>
<td>1.92</td>
</tr>
</tbody>
</table>
From this we see that there is substantially more data available in this investigation than in either of the previous two graduation attempts (although the White male lives the amount of exposure in the 15-29 age range has, in fact, fallen).

3. THE METHOD OF GRADUATION

Initially the intention, in deciding on a method of graduation was to review the methods used to graduate past standard tables in this country and the UK, USA and Australia, and to choose the one we thought most appropriate to our circumstances (bearing in mind that at the end of the day many methods might reduce rates, which for all practical purposes, are exactly the same). A brief summary of these methods appears in Appendix 2.

However, in the process of studying the Whittaker-Henderson method (used to graduate the US life tables) in more detail (London, 1985, Verrall, 1993 and Taylor, 1992) it became apparent that spline graduation was just as acceptable and under certain circumstances could prove to be superior to both parametric and Whittaker-Henderson methods of graduation. Since we had access to a program which could fit cubic spline curves by optimally (i.e. minimum χ²) selecting the knots for a given number of knots we opted for spline graduation.

4. GRADUATION OF THE ALL MALE AND WHITE MALE COMBINED ULTIMATE DATA

The data collected by the CSI Committee cover ages 15 to 114. However, initial attempts at graduation suggested that the data in the age ranges 15-21 and over 90 were too scanty to give useful observations and thus the curves were fitted to data over the age range 22 to 90.

For comparison purposes the White male combined ultimate data were also graduated.

This program has been created by Gerard Farmar as part of the work towards the MBusSc degree.

Details of these graduations appear in Appendix 3. In each case a 3-knot graduation produced a very good fit. In the case of the White male graduation the rates for ages 22 and above were, somewhat arbitrarily, blended into those of the White Male South African Life Table for 1985 (CSS, 1987) to give an estimate of the shape and level of the accident hump.

The results of these efforts appear in Figures 1 and 2 below, and are compared with the 1956/62 and 1972/77 curves in Figure 3.

![All Male Combined Ultimate Graduation Graph](Figure 1: All male combined ultimate graduation)
Figure 2: White male combined ultimate graduation

Figure 3: Comparison of the all male and White male graduations with those of SA56/62 and SA72/77.
From these figures we can note the following:

1. The all-males rates are significantly higher than those in the existing standard tables until age 40. In addition the problems of scanty data at the young and old ages mentioned above are apparent.
2. There has been a flattening of the accident hump as well as an increase in the peak for the White male curve. This may be due, in part, to a blurring of racial distinction in that some offices no longer record population group.
3. Apart from the accident hump White male mortality has improved since 1972/77.

However, before proceeding further with the graduation it is necessary to examine more thoroughly the reasonableness of basing a new standard table on the aggregate male data.

5. THE APPROPRIATENESS OF THE ALL MALE GRADUATION

In order to decide whether or not it is appropriate to aggregate all the male data (medical and non-medical combined) we tested in turn whether or not the rates for each of the following sub-populations differed from those of a "core population" (White male non-medical lives): White male medical, Indian male combined, Coloured male combined, and Black male combined. Details of these efforts can be found in Appendix 4.

Essentially it can be considered to be acceptable to combine two sub-populations even when the rates are statistically significantly different provided that the proportions those sub-populations represent of the whole are not expected to change significantly over time. Thus in deciding whether the rates of the sub-populations differed we examined first whether they were statistically different and then, if they were, whether the sub-population could be expected to increase or decrease substantially in future relative to the whole.

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(a) White male medical vs White male non-medical

As has been the case with many other life assured graduations (e.g. previous South African investigations and the AM(80) graduation in the UK) the difference between the medical and non-medical rates was found to be statistically significant. However, in this case, rather than the medical rates being consistently lower than those of non-medical lives the medical rates at ages below 42 were greater than the non-medical rates, but most of the significant difference occurred above age 59.

Despite these statistically significant differences it is usually the practice to combine medical and non-medical experience. This is done for a number of reasons, chief amongst them being that there is no call for differentiated rates and that the rationale behind medical underwriting at standard rates is, presumably, to produce a group of lives with mortality experience close to that of standard lives. In addition to this the difference in the rates in this experience for ages 20-59 was less than 5%.

For these reasons it was felt that the medical and non-medical rates could be combined.

(b) Indian males combined vs White male non-medical

Indian male combined rates were statistically significantly higher (on average 33% higher) than those of White male non-medical lives. However, since the Indian assured lives account for a small proportion of the total exposure (less than 3%) and since this proportion is unlikely to increase in future inclusion of these lives in an all male experience will not result in any noticeable distortion of the overall rates. Thus it was decided that these rates could be combined with the White male combined experience.
Coloured male combined vs White male non-medical

Coloured male combined rates were also highly statistically significantly higher than those of the White male non-medical lives, but in this case the rates were on average 21% higher than those of the core population. However, in the interests of producing a non-racial standard table it could, conceivably, be argued that although Coloured assured lives accounted for a higher proportion of the exposure (5.6%) than did Indians this proportion was not likely to change very rapidly in future, and hence it was decided that this experience could be combined with that of the White and Indian combined experience.

Black male combined vs White male non-medical

As might have been expected the Black male combined rates are not only highly statistically different from the White male non-medical rates but the difference is also of practical significance. Although the Black assured lives experience represents only a small proportion of the total experience in this investigation (less than 8%) this proportion can be expected to grow rapidly in the foreseeable future. Thus it has to be concluded that it would be neither statistically nor practically acceptable to produce a standard table for all male assured lives combined.

Projected all male combined mortality

To illustrate the effect of the changing mix of the heterogeneous sub-populations we projected the Black and non-Black mortality for 5, 10 15 and 20 years into the future. (Details of the graduation of the Black male mortality rates appear in Appendix 5.) These projections ignored any improvement in the mortality of the sub-groups as well as any extra mortality due to AIDS and assumed that the growth rates in the exposed to risk exhibited by each sub-group over the 1985-90 period would continue in future. (Over this period the Black experience grew by between 10 and 15% p.a. - depending on the age - over most of the age range while the non-Black population grew by between 0% and 5% p.a.)

We had originally intended to check the reasonableness of this assumption by comparing the 5-year projection with the rates obtained from the 1991-94 investigation (CSI Committee, 1996) but unfortunately this investigation excluded the so-called “minimum life cover” policies taken out as investments with a minimum life cover to avoid tax on the proceeds. These policies appear to account for at least 15% of the total experience and can be expected to experience lighter mortality than the remainder of the lives making corroboration impossible. (The comparison indeed shows the 1991-94 rates to be significantly higher than our projections in the 25 to 40 year age group, but part of the difference can probably be explained by higher growth in the number of Black policyholders than projected.)

The resulting weighted average mortality curves compared with that of the all male graduation and SA56/62 appear in Figures 4 and 5 below. (With the exception of the 20-25 age range the SA72/77 rates are very similar to those of SA56/62.)

From Figure 4 we can see that as time progresses there is a flattening out of the curve in the 20 to 45 year age range at increasingly higher levels. Above age 45 SA56/62 will remain conservative for valuation purposes but below this age it will become increasingly too light.

As far as the select rates are concerned (their derivation is discussed in the next section) the effect is more pronounced at the older ages (owing to higher growth rates for durations 0, 1 and 2 exposure than for the ultimate exposure) but less pronounced at the younger ages (due to the fact that the Black and White select rates are closer at these ages). The SA56/62 select rates are too light throughout the age range. This is illustrated by Figure 5 for duration 0.
Figure 4: Comparison of projected all male ultimate rates with those of SA56/62

6. SELECT RATES

We decided to use a simple approach to produce the select rates essentially because the scantiness of the data allows for a wide variety of acceptable curves with none being more preferable than the others. The approach chosen was that
simple linear regression of the select rates against the graduated ultimate
tes to help ensure that the correct order of rates (in particular that
(_x < q(_x+1)_x+2 < q(_x+2)_x+2 < q(_x+3)_x+3) was obtained. Details of these efforts
be found in Appendix 6.

White male combined

The reasonableness of this approach as far as White male combined
data is concerned can be seen from the Figure 6.

As can be seen from Figure 6 there is a fair amount of evidence of
reverse selection between ages 20 and 40. This is not a new
phenomenon (it was observed up to age 30 in both the SA56/62 and
72/77 investigations but was dismissed as being due to random
fluctuation with the pattern of select rates at the younger ages being
extrapolated from that at the older ages). Although it is by no means
clear what might be responsible for this feature, its extension to age 40
could be the result of the inclusion of policyholders from other
population groups in the White male experience by some life offices2.

It doesn't seem appropriate to ignore a feature of the data which has
persisted for over 30 years but on the other hand it makes little sense to
build the reverse selection into the select rates. In the end we decided
to set the select rates equal to the ultimate rates at the same attained
ages in this age range. In order to avoid the discontinuity in the rates
between ages 39 and 40 we regressed \( q(x+1) = q_2(1 + b(x - 40)) \).

Since the regression for durations 1 and 2 were very close to one
another (and since the crude rates even crossed over at some ages) it
was decided to combine durations 1 and 2 to estimate one rate for both
durations.

Details of these regressions can be found in Appendix 6.

1 One possible explanation investigated was that the effect could be the result of
bias in the exposed to risk calculation. This bias was found not to be material
(see Appendix 7).
Black male combined

A similar method was applied to the Black male combined data although, as can be seen from the figure below the data is far more scanty. In addition there appears to be no evidence of reverse selection at the younger ages and the linear trend is upward rather than downward with age.

RATIO OF CRUDE SELECT TO GRADATED ULTIMATE RATES: BLACK MALES

![Graph showing the ratio of crude select to graduated ultimate rates for Black males.]

The scantiness of the data meant that not only were the select rates greater than the ultimate rates at some of the older age groups but the rates for durations 1 and 2 crossed over one another. As a result it was decided to set the rates for durations 1 and 2 equal to one another from age 42 onward and to set this rate equal to the ultimate rates from age 52 onward.

(c) All male combined

For completeness the same exercise was carried out on the all male data. As could be expected the crude rates were very similar to those of the White males and hence so were the resulting graduated rates.

(d) The need for a longer select period

Having discussed the construction of the select rates we now come to the question of whether there is a need for a longer select period. From the White male and the All male curves we note that the gap between the graduated curves for duration 1 and 2 combined and for the ultimate experience gets wider as the ages increase. The gap is quite wide for the older ages which could indicate a need for a longer select period. However, for the Black male graduations the differences between duration 2 and ultimate experiences are fairly small. In fact at the older ages we use the ultimate rates for both durations 1 and 2. Since we expect there to be a much greater proportion of black assured lives making up the total population of assured lives in the future there doesn’t seem to be much point in extending the select period.

7. FEMALES

For completeness we compared the female crude rates against the White and Black male rates graduated above, the data being too sparse to warrant graduating the female rates independently.
ith the exception of the accident hump age range the ratios of the White male to male rates for quinquennial age groups above about age 30 were fairly closely clustered around 40% for durations 0 and 1, and 50% for duration 2 and later.

or Black females the ratios ranged, fairly consistently between 30 and 35% for all durations for ages from the late 20s.

Combining all the lives suggests that the overall ratios in these age ranges could be something like 35% for durations 0 and 1 and 45% for duration 2 and later.

9. CONCLUSION: SOME QUESTIONS FOR THE PROFESSION

he most obvious conclusion that can be drawn from this research is that it does not appear to be appropriate to produce a single standard table on the basis of the aggregated data since the proportions of the heterogeneous sub-populations can be expected to change significantly in future.

Addition to this the appropriateness of the existing standards is also questionable, in particular we must question the conservativeness of SA56/62 the prescribed valuation basis.

uth Africa is a very heterogeneous society and this research once again dictates the dangers of trying to account for demographic features on an aggregate basis (and of blissfully following the assumptions and patterns of experience gleaned from more developed and homogeneous societies). However, rather than segregating further, lives assured data capturing has indeed the opposite direction in that, for various reasons, data is no longer plotted on the basis of “population classification”. It is thus imperative, if the profession wishes to produce useful mortality data in future that a new way of viewing the data be found (income/wealth is one alternative - which may prove difficult - geographical location (e.g. postal code/phone no.) is an alternative which we think has potential).

Following on from the above there is a need to investigate the impact of a worsening aggregate mortality experience on premium and valuation bases and methods of calculation.

There are a number of additional questions arising from this research which could usefully be debated by the profession, for example:

1. Why is there reverse selection in White males in the age range 20-40 years?
2. Why are the rates of medical lives higher than those of non-medical lives for White males in the same age range?
3. Why are the 1991-94 rates greater than the all male projections in roughly the same age range? Is it due to the removal of minimum life cover policies from the more recent investigation, or are the growth rates assumed in our projections too low, or could some of it be AIDS related?
4. The method we used to derive growth rates over the period also allows us to estimate lapse rates. Should such research take place, and should the findings be published (or at least be made available to contributing offices)?
5. Fewer than half of the life offices participated in the investigation (and some of those who did had to be asked to correct the data they submitted). Might it not be considered unprofessional practice not to participate conscientiously in such investigations?
6. Is spline graduation suitable for such graduations and if so how should one go about optimising the fit for smoothness?
7. To what extent is the flattening of the accident hump in the White male experience due to inclusion of lives from other population groups?

ACKNOWLEDGEMENTS

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## APPENDIX 1: THE DATA

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## APPENDIX 2: SUMMARY OF METHODS USED TO GRADUATE OTHER EXPERIENCES

### USA (1975 - 80 Basic Tables)

These tables were based on a 15 year select period and on medical, paramedical and non-medical policies combined.

Separate graduations were performed for males and females.

#### Male graduation

The ultimate rates were graduated using a Whittaker-Henderson type A (i.e. the weights being 1 for all x) method with c, the relativity constant, equal to 2. The smoothness was measured with second-order differences. The rates at ages 15 and 16, and 85 and over were adjusted to make them more acceptable.

The select male rates were graduated using the same method. The value of c was varied by issue age group depending on the smoothness of the crude rates and the volume of data available. The emphasis on smoothness increases with the value of c. c = 0.5 was used for issue ages 14 and under where the curve had special characteristics; c = 1 for ages 15 - 59 where a considerable volume of data was available; c = 2 for ages 60 - 64 and c = 3 for the highest 2 issue age groups where the data was fairly scanty.

The rates were then adjusted to ensure that $q_{x|x+k|+} \geq q_{(x+k)|x|+}$ and $q_{x|x+k|+} \geq q_{(x+k)|x|+}$.

#### Female graduation

Less data was available for female lives than for male lives and in order to achieve the desired smoothness c was set to 3 for both ultimate and select. For even greater smoothness a curve of constant second differences was fitted above age 70. Certain empirical adjustments were also made to improve fit and smoothness for the select graduation.
\( (AM(80) \& AF(80)) \)

In UK started the graduation process by using signs, runs and \( \chi^2 \) tests to compare the crude data of durations 0, 1, 2, 3, 4 and 5+. This was first done for population which combined medical and non-medical lives. Each duration and various groups of durations were compared with each other.

Although comparison of medical with non-medical rates at each duration owed the difference to be significant (medical mortality was consistently over than non-medical mortality) they were combined, as had been done in the past.

After examining mortality by duration the committee graduated, separately, the curves for durations 0, 1, 2, 3, 2 - 4, 1 - 4, 2+ and 5.

Graduations were performed by fitting a parametric curve to the force of mortality using maximum likelihood to estimate the parameters. The Gompertz Makeham family: \( GM^{2.2}(x) = a_0 + a_1 t + \exp\{b_0 + bt\} \) with \( a_0 = \left( \frac{x-30}{36} \right) \) was used for all assured lives graduations.

All calculations the deaths and the exposed to risks at each age were divided by the variance ratio to take account of duplicate policies.

**Australia (IA-8590 Ultimate)**

The Australian assured lives table was graduated using a WHO package called FETIME. The program estimates \( \mu_x \) values at pivotal ages and from these intermediary values were obtained by log-linear interpolation. A Gompertz curve was fitted to the observed rates above age 85.

**South Africa (SA56/62 & SA72/77)**

Graduations were performed by fitting parametric curves using maximum likelihood to estimate the parameters. The formula used for the SA72-77 assured lives table was of the form \( \frac{q_x}{P_x} = Bc^x + De^{-E(lnx-lnx)^2} \). The formula used for the SA56-62 assured lives table was of the form \( q_x = A + \frac{Bc^y}{Dc^y + 1 + Ec^{-2y}} \) where \( y = (\text{pivotal age}) - x \).
APPENDIX 3: THE GRADUATION OF THE ULTIMATE RATES

II male assured lives combined

curves productions with 2, 3, 4 and 5 knots of the data in the age range 22 to 90 were produced and again the 3-knot curve was chosen as being the best. This graduation is shown in Figure 2.

he following tests of adherence to data were performed.

he $\chi^2$ test:

he chi-square test statistic of 75.45511 is less than 77.93049 the chi-square critical value for 59 (= 69-[2(3)+4]) degrees of freedom (5% significance). Therefore accept H0.

aving accepted this test it is important to perform further tests since there are certain features of the graduation that the $\chi^2$ test does not take into account, namely, the existence of a large positive or negative cumulative deviation over part of the age range, an excess of positive or negative deviations, any clumping of deviations of the same sign, and a large deviation which is counter-balanced by a large number of small deviations. Thus the following tests were performed to test each specifically.

he cumulative deviations test:

Whole age range: -0.01084 > -1.96 accept H0
ages 22 - 40: 0.292954 < 1.96 accept H0
ages 41 - 60: -0.21648 > -1.96 accept H0
ages 61 - 80: 0.375437 < 1.96 accept H0
ages 81 - 90: -0.81878 > -1.96 accept H0

his test shows no evidence of positive or negative cumulative deviations over any part of the age range.

The signs test:

| negative: | 36 | z(pos): | 0.361158 accept H0 |
| positive: | 33 | z(neg): | -0.36116 accept H0 |
| 69 |

It is clear from these results that there is no evidence of an imbalance between the number positive or negative deviations.

Serial correlations test:

$r_1 = -0.09284$
$z(r1) = -0.77122 > -1.96$ accept H0

Thus there is no evidence of any correlation between successive ages, i.e. the shape of the graduated curve is the same as the shape of the curve for the true underlying rates.

The standardized deviations test:

| std dev expected cumulative frequency frequency |
|-------|-------|------------------|
|       | frequency | frequency |
| < -3 | 0.09315 | 0 | 0 |
| -3 to -2 | 1.4766 | 1 | 1 |
| -2 to -1 | 9.377445 | 11 | 10 |
| -1 to 0 | 23.55281 | 36 | 25 |
| 0 to 1 | 23.55281 | 59 | 23 |
| 1 to 2 | 9.377445 | 66 | 7 |
| 2 to 3 | 1.4766 | 69 | 3 |
| > 3 | 0.09315 | 69 | 0 |

Although more rates fall between 2 and 3 standard deviations from the graduated curve the spread of deviations appears to be normal and we can thus accept H0 on the basis of this test.
Confidence Interval test:

There are 4 ages out the 69 where the graduated rates fall outside the confidence intervals. Although this is greater than the expected number of 3.45, the probability of this under H0 is 0.455 and thus we can accept H0.

We thus conclude that this graduation is acceptable for representing all male mortality for the period 1985 - 90.

White male combined ultimate lives

Because of the unreliability of the observations at the extreme ages it was decided to fit the curve to data in the age range 22 - 90 using spline graduation. Spline graduations with 2, 3, 4 and 5 knots were produced. The three knot curve was chosen on the grounds that it adhered to the data better than the four and five knot graduations. This is shown in Figure 2.

The following describes the various tests of adherence to data.

The chi-squared test:

The \( \chi^2 \) test statistic was 74.85529 which is less than 77.93049 the chi-square critical value for 59 degrees of freedom (5% significance). Therefore accept H0.

The cumulative deviations test:

<table>
<thead>
<tr>
<th>Whole age range</th>
<th>-0.01692</th>
<th>&gt; -1.96</th>
<th>accept H0</th>
</tr>
</thead>
<tbody>
<tr>
<td>ages 22 - 40:</td>
<td>0.19286</td>
<td>&lt; 1.96</td>
<td>accept H0</td>
</tr>
<tr>
<td>ages 41 - 60:</td>
<td>-0.24241</td>
<td>&gt; -1.96</td>
<td>accept H0</td>
</tr>
<tr>
<td>ages 61 - 80:</td>
<td>0.477904</td>
<td>&lt; 1.96</td>
<td>accept H0</td>
</tr>
<tr>
<td>ages 81 - 90:</td>
<td>-0.84977</td>
<td>&gt; -1.96</td>
<td>accept H0</td>
</tr>
</tbody>
</table>

Thus this test shows no evidence of positive or negative cumulative deviations over any part of the age range.

The signs test:

| negative | 35 | z(pos): 0.120386 | accept H0 |
| positive | 34 | z(neg): -0.12039 | accept H0 |

Thus there is no evidence of an imbalance between the number of positive or negative deviations.

The serial correlations test:

\[ r_1 = -0.04845 \]
\[ z(r_1) = -0.40244 > -1.96 \] accept H0

Thus there is no evidence of any correlation between successive ages i.e. the shape of the graduated curve is the same as the shape of the curve for the true underlying rates.

The standardized deviations test:

<table>
<thead>
<tr>
<th>std dev. frequency</th>
<th>0.09315</th>
<th>0.09315</th>
<th>69</th>
</tr>
</thead>
<tbody>
<tr>
<td>expected frequency 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>cumulative frequency</td>
<td>1.4766</td>
<td>1.4766</td>
<td>68</td>
</tr>
<tr>
<td>frequency</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>&lt; -3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-3 to -2</td>
<td>9.377445</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>-2 to -1</td>
<td>23.55281</td>
<td>35</td>
<td>24</td>
</tr>
<tr>
<td>-1 to 0</td>
<td>23.55281</td>
<td>58</td>
<td>23</td>
</tr>
<tr>
<td>0 to 1</td>
<td>9.377445</td>
<td>67</td>
<td>9</td>
</tr>
<tr>
<td>1 to 2</td>
<td>1.4766</td>
<td>68</td>
<td>1</td>
</tr>
<tr>
<td>2 to 3</td>
<td>0.09315</td>
<td>69</td>
<td>1</td>
</tr>
<tr>
<td>&lt; 3</td>
<td>1.4766</td>
<td>68</td>
<td>1</td>
</tr>
</tbody>
</table>
Although there is one crude rate that is more than 3 standard deviation from the graduated curve, the spread of deviations appears to be normal and we can thus accept H0 on the basis of this test.

Confidence Interval test:

Here are 5 ages out of the 69 where the graduated rates fall outside the confidence intervals. Although this is greater than the expected number of 3.45, the probability of this under H0 is 0.2624 which is insufficient evidence to reject H0.

Thus we conclude that this graduation is acceptable for representing White male mortality for the period 1985 - 90.

The accident hump

In order to get some sense of the level and shape of the accident hump the graduated rates were blended into the population rates below age 22. Although comparison with the observed data suggest that the resulting curve may not be high enough or steep enough it was felt that there was little reason to suppose that the peak should be higher than that of the population as a whole or that the assured life mortality at ages 15-17 should be significantly lower than those of the general population.

The core graduation was performed for the age range 20 - 90 (after somewhat arbitrarily adjusting for duplicate policies, as described in section 2 of the
aper) using a spline graduation program where the user chooses the number of
nots and then the program, when graduating, optimises the positions of these
nots. We found that there was very little difference between the rates for the 2,
4 and 5 knot graduations over this age range and thus we chose the 2 knot
urve since it provided the smoothest curve.

The \( \chi^2 \) test statistic value was 75.32497 \(<\ 80.23209\), the \( \chi^2 \) critical value for
1 \((=71-[2(3)+4])\) degrees of freedom (5% significance). Clearly there is
sufficient evidence to reject H0 on this basis.

'cum dev test:

<table>
<thead>
<tr>
<th>Range</th>
<th>Z-score</th>
<th>Accept H0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole age</td>
<td>0.123234</td>
<td>accept H0</td>
</tr>
<tr>
<td>ages 20-40</td>
<td>0.665133</td>
<td>accept H0</td>
</tr>
<tr>
<td>ages 41-60</td>
<td>-0.29679</td>
<td>accept H0</td>
</tr>
<tr>
<td>ages 61-80</td>
<td>-0.28777</td>
<td>accept H0</td>
</tr>
<tr>
<td>ages 81-90</td>
<td>0.898141</td>
<td>accept H0</td>
</tr>
</tbody>
</table>

Thus there is no reason to reject H0 on the grounds of cumulative deviations.

ign's test:

<table>
<thead>
<tr>
<th>Pos.</th>
<th>Neg.</th>
<th>Z-score</th>
<th>Accept H0</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>34</td>
<td>0.356034</td>
<td>accept H0</td>
</tr>
</tbody>
</table>

Even though the distribution is slightly skew (too many values between 1 and 2
standardised deviations from 0 and too few values between 0 and 1 standardised
deviations) it would be incorrect to reject the curve on the basis of this alone,
all the other tests show good adherence to the data. This is supported by the
fact that out of 71 ages only 2 (compared with an expected number of 3.55
deviations) fell outside the 95% confidence interval containing the true
underlying rates of white male non-medical mortality.

We thus concluded that this graduation fits the crude data well.

Using this graduation as a benchmark (core graduation) we examined whether it
is statistically or, if not, then perhaps practically acceptable to produce a non-
racial standard table.

In the discussion below we test, in order, the White male medical lives for
duration '3+', the Indian males (medical and non-medical combined) for
duration '3+', the Coloured males (medical and non-medical combined) for
duration '3+' and the Black males (medical and non-medical combined) for
duration '3+' against the core graduation.

White male medical duration '3+' v expected
It is also worth noting that the expected rates fell outside the 95% confidence intervals containing the true underlying rates of White male medical mortality 21 out of 61 times against an expected number of 3.05. However 18 of these points occurred in the age range 58 - 80.

A similar pattern was found in the 1972/77 investigation which showed medical mortality greater than non-medical mortality for ages 20 - 29 and non-medical mortality greater than medical mortality for older ages and yet medical and non-medical experiences have also been combined.

In the United Kingdom (CMI, 1988:15-22) the mortality of medical lives was found to be consistently lower than that of non-medical lives yet they were combined because the offices had no use for separate tables. In addition the data for the other population groups did not distinguish between medical and non-medical lives and it would thus not be suitable to separate them for whites and not for the other groups.

We can thus see that although the two mortality curves are statistically different, it is at the older ages that most of the differences occur, i.e. 60 and over. Since it is reasonable to expect that the proportion of medical to non-medical policy holders will remain stable at these ages in the future, and since the objective of medical underwriting is to accept lives with expected mortality similar to that of the standard, it is reasonable to combine the experiences for ages over 60. Note that the differences in the rates at these ages are still fairly small where medical rates are about 90% of non-medical rates.

For ages 20 - 59 the differences are very small (i.e. the medical to non-medical ratio is 95% although for ages 20 - 39 medical mortality is greater than non-medical mortality) and even though it is likely that there will be a change in the proportion of medical to non-medical lives due to the fact that the limits for deciding to test lives will become lower (e.g. because of AIDS) leading to more lives being tested, this ought not to have a major effect on the mortality rates.

We can thus conclude that even though medical and non-medical mortality rates are statistically different it is more practical to amalgamate the two groups as was done in the past.
Indian males (medical and non-medical) for duration ‘3+’ v. expected

The $\chi^2$ test was highly significant with a value of 156.5 which is greater than the 5%, 56 degrees of freedom critical value of 74.5. For eight out of the 61 ages the expected rate fell outside the 95% confidence intervals, against an expected 35 (all of the values fell outside the lower confidence interval). Under H0 the probability of 8 or more values falling outside the confidence intervals is only 0.0108.

In addition the differences in the rates are large with the Indian mortality rates being about 33% higher than the White male non-medical rates. However, since the Indian assured lives make up less than 3% of the total exposure of all male lives and since this proportion is unlikely to increase significantly in future

Coloured males (medical and non-medical) for duration ‘3+’ v. expected

The $\chi^2$ test was highly significant with a value of 146.0 which is greater than the chi-square 5%, 59 degrees of freedom critical value of 77.9. For ten out of the 61 ages the expected rates fell outside the confidence intervals (all of the values fell below the lower confidence interval). The probability under H0 of 10 or more values falling outside the confidence intervals is only 0.00084.
"confidence intervals test" shows the differences to be more significant in those of the Indian assured lives (mainly because there are more Coloured assured lives making the confidence intervals narrower), but the average difference between Coloured mortality and the core population is smaller (21% vs 33%).

In this investigation the Coloured lives represented less than 6% of the total exposure and since it is unlikely that in the future the proportion of policy holders who are Coloured will change enough to effect the rates of mortality for assured lives combined they may be included. (Although it is possible that there could be a greater proportionate change for Coloured than Indian lives the differences in mortality is less for Coloureds.)


c c(male and non-medical) for duration '3+' v expected

c $\chi^2$ test statistic was highly significant with a value of 1 767.7. In addition this even though the confidence intervals are very wide 38 out of 61 of the expected rates fall outside the 95% confidence intervals. The differences in the rates are thus highly statistically significant.

For most of the age range the Black male rates are much higher than the core rates. However, for ages over 62 the rates fall below those of the core population. Although it is unclear why this should happen one possible explanation is that some dependants are failing to claim on death of the assured.

Once again the Black assured lives population makes up a small proportion of the total number of male assured lives (less than 8% in this investigation), nevertheless this proportion is increasing rapidly. Thus given the large differences in mortality rates between black and non-black assured lives it would be inappropriate to combine the two groups.

For this reason it is clear that it is neither statistically nor practically acceptable to produce a single standard table for all male assured lives combined.
APPENDIX 5: BLACK MALE COMBINED ULTIMATE GRADUATION

produced 2, 3, 4 and 5 knot spline graduations for the 22 - 70 age range. The data for Black assured lives are fairly scanty resulting in the rates being sparsely scattered. As a result it is difficult to produce a smooth curve that adheres to the data. Only the 2 knot graduation was considered to be smooth enough to be acceptable, but even this curve tailed off unacceptably at the older ages.

The chi-squared test:

The \( \chi^2 \) statistic value of 36.4 was greater than 56.9 the chi-square critical value for 41 (=49-(2×2)+41) degrees of freedom (5% significance). Therefore Accept H0.

Having accepted the \( \chi^2 \) test we thus performed the following tests:

The cumulative deviations test:

<table>
<thead>
<tr>
<th>Age range</th>
<th>CI</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole age range:</td>
<td>-0.00033 &gt; -1.96</td>
<td>accept H0</td>
</tr>
<tr>
<td>Ages 22 - 40:</td>
<td>-0.18452 &gt; -1.96</td>
<td>accept H0</td>
</tr>
<tr>
<td>Ages 41 - 60:</td>
<td>0.40713 &lt; 1.96</td>
<td>accept H0</td>
</tr>
<tr>
<td>Ages 61 - 70:</td>
<td>-0.58993 &gt; -1.96</td>
<td>accept H0</td>
</tr>
</tbody>
</table>

There is no evidence of significant positive or negative cumulative deviations over any part of the age range.

The signs test:

<table>
<thead>
<tr>
<th>Category</th>
<th>Count</th>
<th>z(pos)</th>
<th>z(neg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
<td>25</td>
<td>0.142857</td>
<td>0.142857</td>
</tr>
<tr>
<td>Positive</td>
<td>24</td>
<td></td>
<td>-0.14286</td>
</tr>
</tbody>
</table>

There appears to be no imbalance between positive and negative deviations about the graduated curve.

The Serial correlations test:

\[ \rho_1 = 0.044218 \]

\[ z(\rho_1) = 0.367303 < 1.96 \text{ accept H0} \]

The graduated curve appears to have the same shape as the true underlying mortality curve.
Standardized deviations test:

<table>
<thead>
<tr>
<th>d dev.</th>
<th>expected frequency</th>
<th>cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; -3</td>
<td>0.06615</td>
<td>0</td>
</tr>
<tr>
<td>3 to -2</td>
<td>1.0486</td>
<td>0</td>
</tr>
<tr>
<td>2 to -1</td>
<td>6.659345</td>
<td>3</td>
</tr>
<tr>
<td>1 to 0</td>
<td>16.72591</td>
<td>25</td>
</tr>
<tr>
<td>3 to 1</td>
<td>16.72591</td>
<td>43</td>
</tr>
<tr>
<td>1 to 2</td>
<td>6.659345</td>
<td>48</td>
</tr>
<tr>
<td>2 to 3</td>
<td>1.0486</td>
<td>49</td>
</tr>
<tr>
<td>&gt; 3</td>
<td>0.06615</td>
<td>49</td>
</tr>
</tbody>
</table>

are more graduated rates between -1 and 1 than we expect. However, we have already accepted the curve on grounds of smoothness the roughness of the distribution is of little concern.

Confidence Interval test:

are only 1 out of the 49 ages where the graduated rates fall outside of the confidence intervals. This is less than the expected number of 2.45. Clearly we accept H0 for this test.

us even though there is some concern as to the shape of the graduated curve decided to accept it.

accident hump

though there is not a great deal of evidence of any accident hump a slight mp resulted from the blending of the graduated rates with the estimates of the population mortality produced by Dorrington, et al (1993). Once again there is evidence that the graduated rates should be lower in the 15-20 year age range then these rates would have been little different from the White male rates wouldn't seem reasonable.
APPENDIX 6: SELECT RATES

White males

It can be seen from Figure 6 that there appears to be some reverse selection with ages 20 and 40. This reverse selection is not an uncommon feature in 1st African assured lives mortality experiences, both SA1956/62 and 1972/77 showed reverse selection for ages less than 30 (MS Committee, 74 and 1983). In addition to this the rates at different durations also stagger each other at some ages.

I am not sure why there is reverse selection, but one possible explanation for extension to age 40 is that some life offices were unable to subdivide the data across the years. As a result of this the White male experience includes assured lives in other population groups with higher mortality especially at the younger ages and earlier durations.

The 1956-62 investigation this feature was attributed to random fluctuation in the pattern of select rates at the older ages were extrapolated to the younger ages. In the 1972-77 graduation no comment was made about this feature in the pattern was similarly extrapolated.

I decided not to follow a similar procedure as this does not reflect the true underlying experience. We also found that the differences between duration 0, 1, 2, and 3+ for ages less than 40 were small. However, because it would be correct to allow for reverse selection in the graduation we set the select rates to be equal to the ultimate rates for durations 0, 1 and 2 for these ages.

For ages over 40 there appears (as can be seen from Figure 6) to be a relationship between the ratios and the ages to which they relate, i.e. \( q_{x} = q_{x} [1 + b_{x} (x - 40)] \).

First regressed for durations 0, 1 and 2 separately, but found that durations 1 and 2 were very close and also crossed over each other.

We then re-graduated combining the data for duration 1 and 2 (i.e. giving us the same rates for the two durations combined) and to avoid the discontinuity between ages 39 and 40, i.e. \( q_{x} = q_{x} [1 + b_{x} (x - 40)] \) for both durations 0 and 1-2. This resulted in the following curves:

Duration 0: \( q_{x} = q_{x} [1 - 0.01558 (x - 40)] \)
Duration 1 & 2: \( q_{x} = q_{x} [1 - 0.01159 (x - 40)] \)

The value of the F-statistic in the regression was highly significant with a p-value of 0.002213 for duration 0 and 0.005818 for duration 1 and 2 and the t-statistics for \( b_{x} \) were also highly significant with values of -12.9864 and -11.5425. The R-squared were high (0.79 in the case of duration 0 and 0.71 for durations 1 and 2).

For duration 0, ages 40 to 80, only 1 out of 41 versus an expected number of 2.05 values fell outside the confidence intervals. For duration 1 and 2 combined only 4 out of 41 values fell out of the confidence intervals. Under H0
checked male select rates

On Figure 7 we can see that there appears to be an upward linear trend for
ations 0 and 1. Duration 2 also shows a linear trend but the data is so scanty
that the relationship is not as clear.

As clear from this and the table above that there is not much evidence of any
reverse selection for duration 0. It was thus decided to fit
\[ q_1 = q_x (a_0 + b_0 \cdot x) \]
over the age range of 22 - 70. Despite the sparseness of the data the regression produced a fairly good fit (p-value of the F-statistic was 0.04661 with an R-squared of 0.65) to the following equation:

\[ q_1 = q_x (0.46786 + 0.005692x) \]

duration 1 and 2 there was evidence of reverse selection for ages over 50.
is is probably due to the scantiness of the data for these ages. The differences between the ultimate rates and the rates for duration 1 were very all for ages over 52. The duration 2 rates for which the data are extremely cross over the ultimate rates 3 times for ages over 52. Thus for ages ≥ 52 we set the duration 1 and 2 rates equal to the ultimate rates. For the younger ages we used the same regression approach as that used for duration 0. is produced and very good fit (p-value of the F-statistic of 0.00623 and R-squared of 0.87) for the following curve for duration 1:

\[ q_{x-1|x+1} = q_x (0.328114 + 0.012923x) \]

same approach for duration 2, however, resulted in a very poor fit as a ult, particularly of the wide dispersion at the older ages.

\[ q_{x-2|x+2} = q_x (0.870763 + 0.000269x) \]  In addition the duration 2 curve passed over the duration 1 curve at age 42 and ultimately the curve of ultimate. Thus in order to avoid anomalies and because the duration 2 rates were significantly higher than the duration 1 rates at the younger ages it was decided use this regression equation for the rates up to age 42 where this curve crossed that for duration 1. Over this age the duration 2 rates were set equal to those for duration 2.

All male select rates

As could be expected most of the features found in the White male assured lives were apparent in the all male select rates.

The same method of graduation were used here i.e. \[ q_{x-[x-\ell]x+\ell} = q_x (a_x + b_x \cdot x) \] for \( x > 39 \) and duration 1 and 2 were combined, except that we did not need to adjust the rates to ensure that they would be equal to the ultimate rates at 40 (at age 40 duration 1-2 rates were equal and duration 0 they differed only in the 5th decimal place). The following were the resulting curves:

Duration 0:

\[ q_{x+1} = q_x (1.555 - 0.0144x) \]

Duration 1 & 2:

\[ q_{x-[x-\ell]x+\ell} = q_x (1.446 - 0.01095x) \]

As might be expected these curves were very good fits with, in the case of duration 0 a p-value of the F-statistic of 0.000314 and R-squared of 0.9, and in the case of the other durations 0.00052 and 0.88 respectively.
Appendix 7: Bias in the Estimation of the Exposed to Risk

Initial crude mortality rates were estimated as follows:

\[ q_{x,t} = \frac{\theta_{x,t}^y + \theta_{x+1,t}^y}{P_{x,t}^y + \theta_{x,t}^y} \]

where \( P_{x,t}^y \) represents the number in force aged \( x \) nearest birthday and curtate duration \( t \) at the end of census year \( y \), and 
\( \theta_{x,t}^y \) and \( \theta_{x+1,t}^y \) represent the number of deaths occurring in census year \( y \) of lives classified as curtate duration \( t \) at date of death and aged \( x \) nearest birthday at the census date following death but where the death occurs after and before the policy anniversary in census year \( y \), respectively.

This implies a policy year rate interval starting on the policy anniversary where duration is exact and lives are aged \( x \) nearest at the next census, and that the initial exposed to risk is estimated by the denominator. This effectively assumes that the exposure of lives which withdrew in year \( y \) after the policy anniversary on which they were \( t \) exact which should have been included in the calculation is equal to that lost by withdrawals in year \( y+1 \) which occurred prior reaching duration \( t+1 \) (which is included in \( P_{x,t}^y \) but should have been included from the initial exposed to risk). As such, at the earlier durations at least, withdrawals in the first half of a policy year are likely to significantly exceed those in the second half this is not a valid assumption. In effect the initial exposed to risk is understated and hence timates of rates are biased upward. However, investigation of the lapse pattern suggests that at worst (i.e. duration 0) this effect is unlikely to aggregate the rate by more than 0.5%.
<table>
<thead>
<tr>
<th>x</th>
<th>q(x)</th>
<th>q(x)+1</th>
<th>q(x)+2</th>
<th>q(x)+3</th>
<th>age x+3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0001721</td>
<td>0.002349</td>
<td>0.003149</td>
<td>0.003937</td>
<td>45</td>
</tr>
<tr>
<td>1</td>
<td>0.002888</td>
<td>0.003149</td>
<td>0.003389</td>
<td>0.003939</td>
<td>46</td>
</tr>
<tr>
<td>2</td>
<td>0.003096</td>
<td>0.003389</td>
<td>0.003585</td>
<td>0.004229</td>
<td>47</td>
</tr>
<tr>
<td>3</td>
<td>0.003317</td>
<td>0.003585</td>
<td>0.003951</td>
<td>0.004712</td>
<td>48</td>
</tr>
<tr>
<td>4</td>
<td>0.003562</td>
<td>0.003951</td>
<td>0.004275</td>
<td>0.005168</td>
<td>49</td>
</tr>
<tr>
<td>5</td>
<td>0.003831</td>
<td>0.004275</td>
<td>0.004629</td>
<td>0.005672</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>0.004124</td>
<td>0.004629</td>
<td>0.005015</td>
<td>0.006226</td>
<td>51</td>
</tr>
<tr>
<td>7</td>
<td>0.004444</td>
<td>0.005015</td>
<td>0.005542</td>
<td>0.006832</td>
<td>52</td>
</tr>
<tr>
<td>8</td>
<td>0.004786</td>
<td>0.005542</td>
<td>0.006385</td>
<td>0.007493</td>
<td>53</td>
</tr>
<tr>
<td>9</td>
<td>0.005159</td>
<td>0.006385</td>
<td>0.007363</td>
<td>0.008213</td>
<td>54</td>
</tr>
<tr>
<td>10</td>
<td>0.005555</td>
<td>0.007363</td>
<td>0.008365</td>
<td>0.009081</td>
<td>55</td>
</tr>
<tr>
<td>11</td>
<td>0.005976</td>
<td>0.008365</td>
<td>0.009481</td>
<td>0.010684</td>
<td>56</td>
</tr>
<tr>
<td>12</td>
<td>0.006422</td>
<td>0.009481</td>
<td>0.010623</td>
<td>0.011819</td>
<td>57</td>
</tr>
<tr>
<td>13</td>
<td>0.006897</td>
<td>0.010623</td>
<td>0.011875</td>
<td>0.013004</td>
<td>58</td>
</tr>
<tr>
<td>14</td>
<td>0.007406</td>
<td>0.011875</td>
<td>0.013099</td>
<td>0.014337</td>
<td>59</td>
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DISCUSSION

Professor R E Dorrington (introducing the paper) gave a brief summary of the method and main findings of the paper focusing particularly on the changing mortality environment facing the profession and the need to find a way to disaggregate the heterogeneous experience.

S C Jurisch: When Rob first asked me to open the discussion on his and Steven’s paper, I thought long and hard about what comments could add value to his work, as well as what questions could be asked that the authors’ have not already raised. The reason for the latter is that the biggest unknown remains the data and it is very difficult to draw firm conclusions from uncertain data.

The authors were also not able to access the data of the individual contributing offices, but instead had to rely on the aggregate data. Perhaps in future those graduating the data should have greater access to the raw data.

Still on the subject of data, less than half of the life offices were willing and able to supply data for the period of the gradation, and we know that the situation for the period 1991 - 1994 was not much better. I therefore reiterate the plea of the authors for companies to try and improve this position. This will not only ensure greater representation of the assured population, but will also make the mortality result less dependent on the data of two or three large players.

Turning to the paper itself, it is interesting to note from Figure 3 the extension of the accident hump and its flattening for white males, though as the authors point out, this could be as a result of lives without a racial classification being assigned to the white group by default.

The question of whether sub-populations can be combined in order to obtain an aggregate table is more controversial. If an aggregate table is going to be used for pricing purposes, the combining of sub-populations will allow a cross-subsidisation to occur. This is, however, not the current trend, which is rather
price by socio-economic status - so an aggregate becomes of less use. This particularly true when the proportion that each sub-population makes up of the whole is continually changing - and at a fairly rapid rate.

That is therefore needed for current pricing and valuation purposes are separate tables by socio-economic class, based on data containing the appropriate variables. In the CSI-report the results of data divided into a high and a low sum assured level as a proxy for socio-economic status is given for the first time. This division is not very successful and the profession must give thought to determining what variables should be captured and analysed, if we wish to price accurately and on this basis. This ties in with the whole question of "right to underwrite", plus our pricing and underwriting practices, and is perhaps appropriate that a cold look at the state of our statistics takes place, per the paper by Moultrie and Thomas.

Looking at the combining of the medical and non-medical rates, I agree with the authors' decision to do so. I did, however, find the fact that the medical rates were higher than the non-medical rates for ages below 42 of concern. Is it because of the inclusion of a large number of minimum life cover policies, at is primarily investment products in the non-medical group with resulting self-selected good mortality, or is it that underwriting is not good enough? Gain too little is known about the data to investigate thoroughly.

This is further seen in the reverse selection in the white male combined data between ages 20 and 40. As the authors point out this effect was discussed in earlier investigations, but its persistency cries for it to be investigated further.

As far as the black male combined data is concerned, the fact that select rates are greater than ultimate rates at some of the older ages is interesting. I believe that this is more likely due to a relatively greater under-reporting of deaths at the ultimate durations than anti-selection. The implication of this is that setting template rates equal to the ultimate from age 52 onwards is likely to result in reasonable select rates but understated ultimate ones!

A brief comment on the female rates concerns their ratio to male rates being even lower than one would have expected from overseas studies, especially for the black females. This could partly be explained by the high level of deaths due to accidents and violence in the male population, or may more likely be as a result of female policyholders being on average of a better socio-economic status than the much larger body of male lives. This could mean that female mortality as a percentage of male mortality will rise as a greater penetration is achieved - this again has pricing and valuation implications.

In conclusion I would like to congratulate both authors on their hard work and for producing an interesting paper which raises many relevant questions.

J H van der Linde: This is an interesting and well-written paper. I am sure that it must have been a satisfying project for the authors.

Analyses by race certainly show up significant variations. What would those analyses look like if further split, for example, by socio-economic group and smoker/non-smoker status? Would race then still play such a prominent role? We shall probably never know the answer since race is not to be a factor in current and future investigations.

I would suspect that the different proportions of smokers (possibly with mortality rates as much as double those of non-smokers, according to investigations of assured lives in USA and UK) amongst the different race groups and sexes could possibly account for a fair proportion - certainly not all of it - of the differences in mortality between races. Surveys of smoking habits by race and sex have shown significant variations in proportions smoking in the general population. For example, very few Indian females smoke.

The authors suggest a number of additional questions which could be debated. I comment on two of them:

(1) The Society of Actuaries publishes the lapse studies conducted annually by the Life Insurance Marketing Research Association (LIMRA). I strongly support the view that similar work should be done here, based on the data that is after all already available. The results of the work should be published to the profession, as is done in North America. First year lapse rates there on whole life policies (just to give an example) are of the order of 17% to 18%. I trust that we
are not scared of "what people might say" about South African lapse rates that are higher, hence hesitate to do and publish the work. What does this conference say?

It is of course very easy to calculate crude lapse rates on a company by company basis from the returns submitted to the Financial Services Board, which are public information. Similarly crude lapse rates for companies grouped by size can be determined by anyone with access to the LOA annual statistical surveys.

The advantage of the "actuarial" approach is that lapse rates can be determined separately for "first-month lapses", "second-month" lapses, and so on.

Lapse and other voluntary termination rates are essential inputs to determining the profitability of the business that is written and would be useful for benchmarking standards against which companies could measure themselves internally.

The less than complete participation in investigations:

The question could well be asked - in this age of consumerism, should there be any co-operative investigations at all? Are those monolithic insurers and their actuaries not just getting together - in cahoots - to pool and share information which could be used to add to their already substantial bargaining power against applicants for life insurance? I pose this question in the spirit of the paper by Moultrie and Thomas discussed earlier today.

Professor R J Thomson: It would have been helpful if the exposed-to-risk nd deaths had been shown by age for each of the graduations. Perhaps this information could be made available on the Internet.

t is a pity that data were only available in aggregate form. For the purposes of the annuitants mortality investigation, data are being obtained from each office separately and there does not seem to be a problem. For that investigation, individual life data are not being obtained. The data consist of total exposure to risk and total deaths, by number and amount, for each category-age combination for each calendar year.

Professor A Asher (written comment): The authors ought to be commended for their courage in attempting to make sense of the data presented to them. It is no credit to the profession - or the industry - that more than half the offices are either incapable, too idle or too selfish to contribute their data. Being denied access to the original data could only have added insult to injury.

The data they have is clearly suspect - in particular, the drop off in black mortality rates at higher ages appears indicates a significant misstatement of ages, or large numbers of unclaimed benefits. (This is not unlikely given that most beneficiaries are probably illiterate.)

Given these questions about the data, the success of a simple 2 knot spline curve raises its own questions. I suspect this arises from the adjustment of the data by the UK variance ratios. There can be no doubt that there are many duplicates amongst the white males (8 million exposure years from a total working population of 1.2 million over 6 years). However, the increase in the variance that would have arisen from the duplicates, may be removed by the reduction in variance due to the heterogeneity in the population. In 1990 the Continuous Statistical Investigation Committee reported that the Redington Michaelson test failed to find evidence that the variances needed adjustment.

Apart from this, the authors appear not to have deducted any degrees of freedom for estimated parameters before performing their $\chi^2$ tests. Benjamin and Pollard also point out that an unknown number of degrees of freedom have to be deducted if "pivotal ages are chosen after an examination of the crude data." As it appears the program they used makes a thorough trawl of the data before choosing the knots, I suggest that a goodly number of degrees of freedom are lost.

Together this will mean that the curves they have fitted are less likely to pass the graduation tests. This should, however, not surprise given the likely imperfections in the data and the heterogeneity of the population.
This brings me back to the point of literacy - or rather education. The question whether we believe that race or socio-economic factors are more important in determining mortality differentials. I suggest at race is a surrogate variable for income and education, and that the latter are ranging so fast that investigating data by race will often lead to misleading results. This does not mean that we should necessarily abandon race classification, but that if we must choose between collecting data on race or on education and income, it seems to me that there could be no contest.

The South African Institute of Race Relations 1994/95 survey shows the proportion of the Black population with matric rising from 0.1% or 9,000 people in 1960 to 4.4% or over 1.2 million in 1991. (30,000 in 1970 and 140,000 in 1980.) The figure is now well over 2 million. For practical purposes, Blacks over 45 can be considered not to have finished school, while most adults under 25 will have done so. This is not to endorse the standard of education, but to point out that enormous changes have occurred in the past 20 years.

Changes in wage levels have also been considerable. The 1995/1996 survey shows that Black wages have risen 50% relative to white wages in the manufacturing sector since 1975. The improvement has been more marked in mining and government. In the 1994/95 Survey average monthly earnings in Transnet are reported as R3,762 for Blacks and R3,798 for whites.

One would hope that this would lead to Black mortality coming closer to white mortality at younger ages. The slow rate at which young blacks are absorbed into the formal economy would inhibit this convergence, but insured lives would presumably be employed. The data we have shows the opposite. Either the theory or the data requires re-examination: you can guess my opinion.

Some additional points arising out of the **sessional discussion** of the paper:

1. The authors pointed out that a comparison of the 91-94 experience with the projected rates after five years showed the 91-94 rates to be heavier than the projected rates in the 20-40 year age range by as much as 21%. Various explanations were offered for this, namely:

   1. The exclusion of the “minimum sum assured” policies from the 91-94 experience.
   2. Greater than expected increase in new business amongst high mortality policy holders.
   3. AIDS deaths.
   4. Violence
   5. Selective lapsation by healthy lives.

2. Various suggestions were made for how the experience might be disaggregated into homogeneous subgroups. Among these were the following:

   1. Reintroduce race as a classification criterion.
   2. Geographical (either post code or phone number).
   3. Mode of payment.
   4. Occupation, and/or education level.
   5. By life office.
   6. By “class” as determined by life office using their underwriting procedures.

3. The general feeling was that there was some usefulness to publishing the table(s) as a South African standard (if nothing else as a record of the progression of the mortality experience and to replace the exiting standard).

**Professor R E Dorrington** (replied):

Mr van der Linde mentioned that first year lapses in North America were of the order of 17% to 18%. This is not very different from what we found. However it should be pointed out that the current format of the data (being designed for a
mortality investigation) will not allow for a direct estimation of lapses by month since entry.

Professor Asher commented on the drop off in Black mortality rates at the 7er ages. For the reasons he outlined we do not have a great deal of confidence in these rates and would urge anyone wishing to make use of the data to rather fit a Gompertz curve to the later part of the age range.

Professor Asher also questioned the number of degrees of freedom employed in a Chi-Squared test. The degrees of freedom were calculated using "m-2n+4" where m is the number of observations and n the number of knots. I think that question was based on an incorrect notion of how degrees of freedom work. For certain models, such as linear models with normal residuals, various statistics have distributions (e.g. F, t or Chi) whose degrees of freedom are obtained by subtracting the number of parameters estimated from the number of observations. This has led to a degree of freedom rule, which is a rule of thumb rather than a rigorous rule. If the number of degrees of freedom depends on the method of optimisation, then you would get a different value of degrees of freedom if you produced maximum likelihood estimates using a random guess technique, a Newtonian technique, or by analytical techniques. This seems quite absurd. Besides this, the adjustment is the same as that used by others (e.g. McCutcheon) who have employed a similar procedure.

Turning now to the sessional meetings. The higher mortality exhibited by the 92-94 experience is unlikely to be explained by any one of the causes suggested. However, it is possible that the difference could be explained by a combination of all of these causes. For example, according to the CSI paper, exclusion of minimum sum assured business will only explain some 3% of the mortality, and AIDS and violent deaths are unlikely to have been very significant during this period. In addition, for an increase in new business to be responsible for all of the difference the percentage of the inforce which are back would have had to increase from 10% to 35% in five years, which isn't seem possible. The point is that the aggregate experience has, for whatever reasons, worsened more than anticipated by the projection based on own new business growth rates.

The CSI committee will be investigating the various suggested criteria for disaggregating the experience in the coming year, as well as giving consideration to the possibility of producing a standard table on the basis of the rates in this paper.