EDITORIAL
LONG RUN SE VOET: DEBUNKING THE MANTRA OF THE EQUITY CULT

An oft-repeated mantra of the investment-management world is: “In the long run equities will outperform bonds.” A slightly modified version is: “In the long run equities are expected to outperform bonds.” Sometimes ‘bonds’ becomes ‘cash’ or ‘bonds and cash’. The first version is patently incorrect. The second is misleading. I find these statements embarrassing, especially when they are used by actuaries to justify ‘lifestyle strategies’ for defined-contribution (DC) retirement funds.

As long as 43 years ago, Mossin (1968) proved that, for an investor with constant relative risk-aversion and constant distributions of returns on risky assets over time, her time horizon made no difference to her asset-allocation decision.

Here it should be noted that ‘constant relative risk-aversion’ means constant over the range of outcomes, not over time. Clearly, as time passes, the investor’s relative risk-aversion may change and her expectations may change, and such changes will affect her asset allocation. But at a given time her time horizon does not affect her decision. This is referred to as the ‘myopic’ feature of constant relative risk-aversion. Not all investors necessarily show constant relative risk-aversion. However, such risk-aversion is not abnormal, and any generalisation that disregards investors with such risk-aversion is at best a sweeping overstatement.

Also, to the extent that the annual risk-free return and the distribution of the annual return on the risky asset are affected by the investor’s time horizon, her investment-allocation decision will be affected. If the mantra were qualified by this proviso, it may be valid. Samuelson (1994) and Asher (2007) show how this may arise. In general it is not so qualified.

1 e.g. Ralfe, Speed & Palin (2004)
Samuelson (op. cit.) observed that the myopic feature seemed not to have affected the mindset of investment managers. He stated:

… there cannot be any sure-thing syllogism pushing long-term investors into more equity tolerance than short-term investors;

and:

… once everyone comes to believe in and act on ‘Always hold stocks,’ that tactic will self-destruct in the way that the Tokyo equity bubble self-destructed after New Year’s Day 1990.

That paper served to contradict the historical arguments of the time that equities had invariably outperformed bonds, as propounded in what industry participants have referred to as the ‘Buy-and-hold Bible’ (Siegel, 1994). Nevertheless, such arguments have persisted.

As argued by Ralfe, Speed & Palin (2004):

[It is true]… that the longer the time horizon, the more likely that equities will outperform bonds. This is… often misinterpreted as ‘The risk of holding equities versus bonds decreases the longer the time horizon.’ The ‘long term’ nature of pension funds, compared with other investors, seems to clinch the argument for pension funds holding equities. Pension funds can apparently be rewarded versus other investors for their ability to take a long-term view. This argument is fatally flawed; there is not a free lunch for those with a long time horizon. The risk of an equity portfolio increases over time.

Yet members of the actuarial profession and others who should know better\(^2\) persist in drawing this conclusion.

The mantra is typically applied to ‘lifestyle strategies’ by arguing that, because “in the long run equities will outperform bonds and cash,” younger DC-fund members should be exposed to equities, and because the same will not necessarily be true in the short run, during the last ten or fifteen years before retirement they should gradually be transferred to bonds and cash.\(^3\) Reference to this strategy as a ‘lifestyle strategy’ is both obscure and misleading: it has nothing to do with lifestyle. Presumably it comes from the need to maintain a chosen level of lifestyle after retirement; the mantra would suggest that, if the member does not follow the lifestyle strategy then she will not be able to maintain her lifestyle. But that depends on the level of lifestyle required and the amount of retirement savings available to maintain it, not on the amount of risk that the member is willing to undertake in order to have some chance of maintaining the lifestyle to which she aspires. In fact the latter criterion would necessitate speculative investing instead of the prudence required of trustees in advising or implementing strategies.

This editorial does not purport to represent original research; it merely arises from

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\(^2\) e.g. Templin (2006)

my frustration with the persistence of this mantra despite the literature on the subject. Interested readers are referred to Samuelson (op. cit.) and to the numerous citations of that paper. In the actuarial literature, Ralfe, Speed & Palin (op. cit.), Shen (unpublished) and Carne (unpublished) are of interest.

The unmodified version of the mantra
To consider the unmodified version of the mantra, let us suppose that annual returns on equities are independent and that historical annual returns constitute a sample of such returns. The opposite extreme is that history is a sample comprising one observation, from which we can draw no inferences at all about future returns. Between these two extremes lie models of autocorrelated returns. We shall measure ‘returns’ as annual forces of return in excess of inflation; i.e. as ‘real’ forces of return. Let us further suppose that estimates of the means and variances of returns based on annual returns over the period 31/12/1986 to 31/12/2008 (the period used for Thomson & Gott (2009)) are unbiased estimates of future returns; i.e. that the rational-expectations hypothesis (Muth, 1961) applies. Let us suppose that the total-return index of equities $S_t$ follows geometric Brownian motion. On this basis, at time $t$: 

$$\ln S_t = \mu t + \sigma \sqrt{t} Z;$$

where:
- $\mu$ is the expected annual return;
- $\sigma$ is the volatility of the annual return; and
- $Z \sim N(0,1)$.

From the data used for Thomson & Gott (op. cit.), the estimates of these parameters are:
- $\hat{\mu} = 0.0561$; and
- $\hat{\sigma} = 0.2159$.

For the purposes of comparison with bonds, we use index-linked gilts, on which the real returns to maturity may be derived from the yield curve. For consistency we use the index-linked gilt spot yield curve as at 31/12/2008 to derive these returns. For the purposes of illustration let us assume that forward rates on index-linked gilts remain constant beyond 20 years.

In Figure 1 the mean and 95% confidence limits of $Q_t$, the natural logarithm of the value of equities at time $t$ is compared with that of the value of an index-linked gilt at maturity at that time; in all cases, ‘value at time $t$’ is the value at that time per unit invested at time 0. In fact, since it is essentially the lower 95% confidence limit that is of interest, it is effectively a 97.5% limit. For reasons that become apparent below, the projection is over the very long period of 140 years.

It may justifiably be argued that 140 years is too long a projection period for a model based on 22 years’ data. Furthermore, it may be argued that such a sample may be prone to be influenced by outliers. Other arguments might also be levelled against this discussion: sovereign risk should be allowed for; so should fat tails, mean reversion and model risk; and the assumption of constant forward rates beyond 20 years is questionable. In the
long run, though, the uncertainties of equity expectations are arguably going to be greater than those of yields on index-linked gilts, which would merely serve to strengthen my argument. On the other hand, alternative models could provide lower standard deviations and thus lower uncertainty for equities. The challenge to the cult of the equity is to produce a more credible model that would justify the mantra.

From Figure 1 it may be seen that it will be about 57 years before the lower 95% confidence limit of the return on equity becomes positive. It will be about 87 years before the lower 95% confidence limit of the return on equity exceeds the real risk-free return.

Furthermore, the lower 95% confidence limit will never converge to the mean; although the probability that equities will outperform index-linked gilts increases, the funnel of doubt will continue expanding forever. The same is true of any lower confidence limit.

In the long run equities will not necessarily outperform bonds. As shown by Bodie (1995), even if equity returns are mean-reverting in the long run, bonds may outperform equities; whilst the probability that equities will outperform bonds in the long run tends to 100%, the cost of insuring against equity underperformance increases over time, regardless of mean reversion.

The modified version of the mantra
It is also misleading to state that “In the long run equities are expected to outperform bonds.” What do we mean by ‘expected’? We cannot mean merely that the expected value of the return on equities will be greater than the risk-free return. That would be true in the short term as well as the long term.

Secondly, ‘expected value’ is a technical term meaning the probability-weighted value.

Figure 1: Comparison of values of equities and index-linked gilts
We cannot necessarily ‘expect’, with any reasonable understanding of that word that, just because the expected return is greater, equities ‘will’ outperform the risk-free rate.

Thirdly, if we tell our members that this is a reasonable expectation then, in terms of the Pension Funds Act, we must reserve for it; because of the implicit guarantee that the fund’s equity investments will outperform bonds, we cannot sign a statement that no valuation is required. The guarantee implies that the value of the liabilities is greater than the value of the assets in which members’ interests are invested; the extra liability must be valued for each member according to her/his term to retirement.

Fourthly, even in the absence of any implicit guarantee, this interpretation would be meaningless to a risk-averse member. If we decide on behalf of a member that she must make her investment-channel choice merely on the basis of expected values, without regard for the risks, we are effectively assuming that she is risk-neutral. In so assuming, we are being imprudent. This conflicts with trust law, and we are laying ourselves and our clients open to litigation by members.

So what do we mean by ‘expected’? Do we mean ‘more than likely’? If so, are we merely implying that the probability of outperformance is greater than 50%? In that case the objections of interpretation in terms of expected value apply similarly. (Notice from Figure 1 that the distribution is symmetric about the mean, so that ‘expected value’ and ‘median’ may be used interchangeably. This, of course, follows from the assumption of geometric Brownian motion.) The only logical conclusion is that the probability of outperformance is substantially greater than 50%. What do we mean by ‘substantial’: 90%? 95%?

Figure 2 shows the probability that equities will outperform index-linked gilts.

Figure 2: Probability that equities will outperform index-linked gilts
From Figure 2 it may be seen that, whilst the probability that equities will outperform over a 23-year period is 75%, it is 45 years before the probability reaches 90% and 116 years before it reaches 95%.

It does not follow from Figure 2 that equity risks decrease over time. In fact equity risks increase over time as reflected by the expanding funnel of doubt in Figure 1.

Whether these probabilities are worth the risk depends on the risk-aversion of the member. As usual, ‘relative risk-aversion’ is defined as:

$$r(x) = -x \frac{u''(x)}{u'(x)};$$

where $x$ is the outcome and $u(\bullet)$ is the utility function.

Figure 3 shows the ‘break-even’ constant relative risk-aversion. The break-even level is such that a member with constant relative risk-aversion greater than the break-even level would choose index-linked gilts, whereas a member with lower risk-aversion would choose equities. (Note that, as explained in the introduction, ‘constant relative risk-aversion’ means constant over the range of outcomes, not over time.)

From Figure 3 it may be seen that, over periods up to 25 years, a member with constant relative risk-aversion of more than 2.5 would choose index-linked gilts. For very long time horizons, only members with constant relative risk-aversion less than about 3 would choose equities. Relative risk-aversion levels of 2.5 or 3 are not particularly high. In Thomson (unpublished) it may be seen that, in the sample studied by Levitan (unpublished), about 55% of members showed average relative risk-aversion greater than 2.5 and about 48% showed average relative risk-aversion greater than 3. In

![Figure 3: Break-even risk aversion](image-url)
Thomson (unpublished) it was suggested that, for the purposes of advice to members by
trustees, levels of risk-aversion of about 4 would be appropriate. At that level it would
certainly be advisable for a member to invest in index-linked gilts, regardless of her time
horizon.

In practice, it would be preferable to allow a mix of equities and index-linked gilts,
as this would allow some diversification. But the point remains that the conclusion
inferred even from the modified version of the mantra is wrong: investment in equities
at younger ages is not preferable to investment in index-linked gilts. Lifestyle strategies
are misguided. If the conclusions of this editorial are correct then actuaries reciting
the mantra in either form are acting unprofessionally, as are those who infer lifestyle
strategies from it. The refutation of these conclusions is in the court of the proponents of
lifestyle strategies.

**Ex-ante expectations**
The discussion above is illustrated with reference to historical expected values. The use of
historical values as ex-ante expectations depends on the rational expectations hypothesis.
As Wilkie (1995) argues, long-run historical mean returns on equities cannot be taken as
indicative of ex-ante expectations. In the long run the downward trend in dividend yields
has exhausted itself. In effect, the risk-premium puzzle (Mehra & Prescott, 1985) is
resolved by rejecting the rational-expectations hypothesis and reaching ex-ante expected
returns, which are inescapably subjective, by discussion. As Samuelson (op. cit.) puts it:

![Figure 4: Comparison of values of equities and index-linked gilts: ex-ante expectations](image-url)
Figure 5: Probability that equities will outperform index-linked gilts: ex-ante expectations

Figure 6: Break-even risk aversion: ex-ante expectations

Index-linked gilts preferable

Equities preferable
… keep repeating to yourself firmly: ‘We have only one history of capitalism.’ Inferences based on a sample of one must never be accorded sure-thing interpretations.

As argued in Thomson (2010) in the light of literature cited there, a reasonable ex-ante expected return on equities would be about 0.037 above risk-free returns. Figures 4 to 6 show the results corresponding to Figures 1 to 3 using this basis for ex-ante expected returns on equities. Volatilities remain unaltered; it is assumed for the purposes of illustration that ex-ante volatilities are unbiased.

From Figure 4 it may be seen that it will now be about 78 years before the lower 95% confidence limit of the return on equity becomes positive. It will be about 131 years before the lower 95% confidence limit exceeds the real risk-free return.

Again, no lower confidence limit will ever converge to the mean; the funnel of doubt will continue expanding forever.

From Figure 5 it may be seen that, whilst the probability that equities will outperform over a 16-year period is 75%, it is 56 years before the probability reaches 90% and 93 years before it reaches 95%.

Whereas Figure 3 showed break-even constant relative risk-aversion that varied over time horizon, Figure 6 shows a uniform break-even relative risk-aversion. This is because constant relative risk-aversion allows for myopic decision-making: because of the constant difference between ex-ante expected returns on equities and returns on index-linked gilts the time horizon is irrelevant. From that figure it may be seen that, regardless of the time horizon, a member with constant relative risk-aversion of more than 2.6 should choose index-linked gilts, whilst a member with lower constant relative risk-aversion should choose equities. As Samuelson (op. cit.) stated, on assumptions like those made here:

… it is an exact theorem that ‘The investment horizon can have no effect on your portfolio proportions;

and

… if ever sure-thing dominance tempts you... Restudy the paradoxical case of myopic age-independent prudence.

The paradox is illustrated by the observation above that, after 56 years, the probability that equities will outperform index-linked gilts is 90%, yet at the reasonably modest relative risk-aversion of 2.6 it is still not preferable to invest in equities. Figure 7 resolves this paradox. The expected utility of the outcome is:

$$E\{u(Q)\} = \int_{-\infty}^{\infty} u(q)f(q)\,dq$$

Where \(f(q)\) is the probability density of \(Q\), the natural logarithm of the value at the time horizon of 56 years, and \(u(q)\) is the utility of the outcome \(Q=q\) at that time horizon for constant relative risk aversion of 2.6. Figure 7 shows the utility-weighting of the lower tail of the distribution of aggregate returns for a time horizon of 56 years, i.e.:
The figure therefore enables the reader to see the range of $q$ over which the influence on the expected value is substantial. The figure therefore focuses on the lower tail of the probability density function where the absolute value of $u(q)f(q)$ is substantial. The curves in that figure are not to scale.

On the left-hand side of Figure 7 the value of $u(q)f(q)$ tends to zero as $f(q)$ tends to zero and on the right-hand side $u(q)f(q)$ tends to zero as $u(q)$ tends to zero. The most serious influence on the member’s expected utility arises in the vicinity of $-1.3$, i.e. an average force of return of $-0.023$, giving a value at the time horizon of 27% of the value at time 0—despite the fact that the probability of such a low outcome is only 0.6%. An outcome like this is quite extreme. If the shares listed on the JSE Securities Exchange were to lose 73% of their value in real terms, even after allowance for the reinvestment of dividends, the economy would be in distress. On the one hand one might argue that at these extremes members whose retirement fund entitlements were invested in equities would resort to the deferment of their retirement or to reliance on supplementation from state old-age pensions or family support, so that the expected utility of the retirement benefit alone is an inadequate basis of decision-making at time 0. Also, at these extremes, the effects of fat tails cannot be ignored. One might question whether, under such extreme circumstances, sovereign risk would remain unimpaired in the face of falling government revenues. This would affect the payment both of state old-age pensions and of coupons and redemptions on index-linked gilts. To such long time horizons the question whether

**Figure 7: Utility-weighting of the lower tail of the distribution of aggregate returns for a time horizon of 56 years**
equities are preferable to index-linked gilts is clouded in uncertainty. However, what is certain is that the answer has nothing to do with the mantra.

Other justifications of greater exposure to equities for younger members have been discussed in the literature (e.g. Samuelson, op. cit.), such as diversification from bond-like human capital and opportunities to work more before retirement; but such justifications also have nothing to do with the mantra.

Apart from the extremes that might arise over very long periods into the future, under ex-ante expectations with constant relative risk-aversion and geometric Brownian motion, index-linked gilts are by far preferable to equities, regardless of time horizon, for levels of relative risk-aversion of about four. This not only debunks the mantra, it also makes a mockery of the ‘lifestyle strategy’. Whatever is good for members approaching retirement is equally good for younger members, and vice versa.

**What can we say?**

On the basis of the above discussion, all we can say in favour of equities is that:
- the expected performance of equities is greater than that of index-linked gilts;
- it is more probable that they will outperform index-linked gilts than that they will underperform them;
- the longer the time horizon, the greater is the probability that equities will outperform index-linked gilts; and
- for members with low relative risk-aversion (on our assumptions less than 2.6), equities are preferable.

What we can say in favour of index-linked gilts is that:
- there is no sure-thing syllogism favouring equity investment, so it cannot necessarily be expected that equities will outperform gilts in the long run;
- under constant relative risk-aversion the investor’s time horizon makes no difference;
- for prudent members (with relative risk-aversion greater than 2.6, which on the basis of Thomson (unpublished) should include trustees), index-linked gilts are preferable;
- the time horizons over which the lower 95% confidence limit of returns on equities will become positive, let alone exceed the return on index-linked gilts, is much longer than the expected remaining lifetimes of active members; and
- no lower confidence limit of the return on equities will ever converge to its mean; the funnel of doubt may well continue expanding forever.

As observed in the introduction to this editorial, not all investors necessarily show constant relative risk-aversion. However, such risk-aversion is not abnormal, and any generalisation that disregards investors with such risk-aversion is at best a sweeping overstatement. Also, to the extent that the annual risk-free return and the distribution of the annual return on the risky asset are affected by the investor’s time horizon, her investment-allocation decision will be affected. If the mantra were qualified by this proviso, it may be valid. In general it is not so qualified.

None of the arguments in favour of equities constitutes a convincing reason for
advising the implementation of a lifestyle strategy. For extremely long periods (e.g. the period of 56 years contemplated above) we cannot make any statement about the relative advantages of equities.

This editorial does not advocate 100% investment in index-linked gilts. The portfolio selection problem should optimise the expected utility of returns over all asset classes. For that purpose constant relative risk-aversion may not be appropriate. Also, consideration should be given to the kurtosis of equity returns and to the sensitivity of the optimal portfolio to the parameters of the model. These are matters not of original actuarial scholarship, but of actuarial practice: the arguments have already been made in the literature and the models are available. The challenge to the profession is to debunk the mantra of the equity cult.

RJ Thomson

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REFERENCES